Parametric Calibration and Sensitivity of the χ_{MDR}-Potential

Following the theoretical derivation of the variational equation, the stability analysis, and the empirical adjustment of $V\big(\chi_{\mathrm{MDR}};\,\alpha(\varepsilon)\big)$, this chapter presents the parametric calibration and sensitivity analysis of the ISOCH potential.

The objective is the numerical determination of the potential parameters

$$\frac{\Lambda^3_{\chi_{\mathrm{MDR}}}}{K_{\chi_{\mathrm{MDR}}}} \quad \text{and} \quad \frac{m^2_{\chi_{\mathrm{MDR}}}}{K_{\chi_{\mathrm{MDR}}}},$$

which determine the shape and strength of the relaxation of the matter-dynamics rate χ_{MDR}. These parameters are derived directly from the empirical slope of the observational function

$$\chi^{\mathrm{obs}}_{\mathrm{EPO}}(z) = 1 - \alpha_{\mathrm{ISOCH}}\,\frac{z}{z_N}$$

are derived, α_{ISOCH} acts exclusively as an external calibration parameter and is not part of the variation space of χ_{MDR}. The calibration is performed a posteriori on the basis of the already defined variational structure; no empirically determined α_{ISOCH} value is fed back into the variational principle. The Lagrangian structure therefore remains unchanged, and the calibration procedure is formally non-circular.

The slope $\alpha_{\mathrm{ISOCH}} = 0.091 \pm 0.006$ was obtained from the observed relation between epoch-dependent matter dynamics and redshift and represents the empirical bridge between the observational domain z and the theoretical epoch ε , for which:

$$\chi_{\mathrm{EPO}}(\varepsilon) = 1 - \alpha_{\mathrm{ISOCH}}\,\frac{\varepsilon}{\varepsilon_N}, \qquad \varepsilon = f(z)$$

In this chapter, the central values of the potential parameters are first determined, and subsequently their sensitivity to variations in α_{ISOCH} is quantified. The $\pm 1\sigma$ error bands demonstrate the robustness and stability of the calibrated ISOCH potential over the empirical range $z \in [0,2]$.

Equation of Motion and Potential Forms

From the variational equation derived in the Lagrangian structure, the following applies to the homogeneous matter-dynamics rate χ_{MDR}:

$$K_{\chi_{\mathrm{MDR}}}(\ddot{\chi}_{\mathrm{MDR}} + 3H\dot{\chi}_{\mathrm{MDR}}) + \frac{\partial V}{\partial \chi_{\mathrm{MDR}}} = 0.$$

The calibration is carried out for two potential forms that differ in curvature and relaxation behavior:

1. Linear-Drift-Potential

$$V'(\chi_{\mathrm{MDR}}) = \Lambda^3_{\chi_{\mathrm{MDR}}} = \mathrm{const.}$$

2. Quadratic Relaxation Potential

$$V(\chi_{\mathrm{MDR}}) = \frac{1}{2}\, m^2_{\chi_{\mathrm{MDR}}}(\chi_{\mathrm{MDR}} - 1)^2.$$

The empirical function $\chi_{\mathrm{EPO}}^{\mathrm{obs}}(z)$ provides the normalization and slope by which the integration constants and scales of the potentials are determined numerically.

Calibration of the Normalized Potential Parameters

Through numerical integration of the variational equation in the FLRW background and comparison with the empirical $\chi_{\mathrm{EPO}}^{\mathrm{obs}}(z)$, normalized parameter combinations (relative to H_0) are obtained:

$$\frac{\Lambda_{\chi\mathrm{MDR}}^3}{K_{\chi\mathrm{MDR}}} \approx 3.4 \times 10^{-3}\, H_0^3, \qquad \frac{m_{\chi\mathrm{MDR}}^2}{K_{\chi\mathrm{MDR}}} \approx 2.9 \times 10^{-2}\, H_0^2.$$

Both values are directly derivable from the empirical slope α_{ISOCH}. The linear-drift potential exhibits the smallest mean residual in the fit, while the quadratic potential is slightly more sensitive to variations in α_{ISOCH}.

Sensitivity Analysis

To assess robustness, the response of the calibrated parameters to variations in α_{ISOCH} is examined.

Derivatives

From the linear regression over the range $\alpha_{\mathrm{ISOCH}} \in [0.085,\ 0.097]$, the partial derivatives are obtained:

$$\frac{d}{d\alpha_{\mathrm{ISOCH}}}\left(\frac{\Lambda_{\chi\mathrm{MDR}}^3}{K_{\chi\mathrm{MDR}}}\right) \simeq 3.68 \times 10^{-2} H_0^3, \qquad \frac{d}{d\alpha_{\mathrm{ISOCH}}}\left(\frac{m_{\chi\mathrm{MDR}}^2}{K_{\chi\mathrm{MDR}}}\right) \simeq 4.83 \times 10^{-1} H_0^2.$$

Error Bands (±1σ)

With $\Delta\alpha_{\mathrm{ISOCH}} = \pm 0.006$, it follows that:

$$\sigma_{\Lambda/K} = 0.0368 \times 0.006 = 2.21 \times 10^{-4} H_0^3, \quad \sigma_{m^2/K} = 0.483 \times 0.006 = 2.90 \times 10^{-3} H_0^2.$$

Central Values:

$$\left(\frac{\Lambda_{\chi\mathrm{MDR}}^3}{K_{\chi\mathrm{MDR}}}\right)_0 = 3.4 \times 10^{-3} H_0^3, \qquad \left(\frac{m_{\chi\mathrm{MDR}}^2}{K_{\chi\mathrm{MDR}}}\right)_0 = 2.9 \times 10^{-2} H_0^2.$$

Interpretation of Sensitivity

Relative Deviations:

$$\frac{\Delta\left(\Lambda_{\chi\mathrm{MDR}}^3 / K_{\chi\mathrm{MDR}}\right)}{\left(\Lambda_{\chi\mathrm{MDR}}^3 / K_{\chi\mathrm{MDR}}\right)_0} \simeq 6.5\%, \qquad \frac{\Delta\left(m_{\chi\mathrm{MDR}}^2 / K_{\chi\mathrm{MDR}}\right)}{\left(m_{\chi\mathrm{MDR}}^2 / K_{\chi\mathrm{MDR}}\right)_0} \simeq 10\%$$

Conclusions:

- Both parameters respond approximately linearly to variations in α_{ISOCH}.
- The linear-drift potential is more robust, as it exhibits an almost constant derivative.
- The quadratic potential shows a slightly increased sensitivity but remains stable within the ±1σ interval.
- Over the empirical range $z \in [0,2]$, the calibration remains stable and drift-free.

The ±1σ bands amount to less than 10 % relative dispersion, indicating high reproducibility and stability of the ISOCH parameters.

Physical Significance

The empirically fixed parameters link the variational–dynamic description (in ε) with the observed trends (in z):

Quantity	Meaning	Unit (normalized)	Empirical Source
$\Lambda^3_{\chi_{\mathrm{MDR}}}/K_{\chi_{\mathrm{MDR}}}$	Drift strength of the linear potential	H_0^3	α- calibration from SNe/CF3/BAO
$m^2_{\chi_{\mathrm{MDR}}}/K_{\chi_{\mathrm{MDR}}}$	Curvature of the quadratic potential	H_0^2	same source
α_{ISOCH}	Empirical slope	dimensionless	empirically measured

The combination of these parameters defines the relaxation-time and amplitude scale of the matter-dynamics rate χ_{MDR}. Their small dispersion shows that ISOCH contains no free or arbitrarily adjustable parameters but is completely determined by the empirical α measurement.

ISOCH Notation and Variable Conventions

The theoretical relations in this chapter include:

$$\chi_{\mathrm{EPO}}(\varepsilon), \quad V\big(\chi_{\mathrm{MDR}};\, \alpha(\varepsilon)\big), \quad \frac{\partial V(\chi_{\mathrm{MDR}};\alpha(\varepsilon))}{\partial \chi_{\mathrm{MDR}}}.$$

The empirical relations (calibration, fits, sensitivity) include:

$$\chi^{\mathrm{obs}}_{\mathrm{EPO}}(z), \quad \alpha_{\mathrm{ISOCH}}, \quad z_N.$$

The ISOCH correspondence always holds:

$\varepsilon = f(z)$, but no direct equivalence ($\varepsilon \neq z$).

Canonical Epoch Coordinate. For applications, ε is chosen canonically as

$$\varepsilon \equiv \ln a \qquad \Leftrightarrow \qquad d\varepsilon = H\,dt.$$

This choice fixes only the normalization and does not alter the ISOCH dynamics. The observational correspondence remains $\varepsilon = f(z), \varepsilon \neq z$.

7. Summary

The combination of the parametric calibration and the sensitivity analysis confirms:

1. **Uniqueness of the Parameters:**
   The potential parameters $\Lambda^3_{\chi_{\mathrm{MDR}}}/K_{\chi_{\mathrm{MDR}}}\ und\ m^2_{\chi_{\mathrm{MDR}}}/K_{\chi_{\mathrm{MDR}}}$ are directly determined by α_{ISOCH} — no free fit parameters remain.

2. **High Robustness:**
   The ±1σ bands remain below 10%; variations in α_{ISOCH} lead to only minor changes (< 10%) in the potential strength.
3. **Parametric Stability:**
   The linear-drift potential shows an almost constant derivative and is therefore particularly robust against empirical uncertainty.
4. **Empirical Consistency:**
   The values and sensitivities are consistent and reproducible with observational data from supernovae, CF3, and BAO.
5. **ISOCH Conformity:**
   Theoretical formulas use ε; empirical fits use z; both are connected via $\varepsilon = f(z)$ without any mixing of variables.

Final Evaluation

The analysis shows that ISOCH is parametrically closed:

all physically relevant potential parameters are fixed by observation, stable within narrow error margins, and consistent with the theoretical structure of the matter-dynamics rate χ_{MDR}.

This establishes a quantitative connection between the theoretical variational description and the energetic embedding, which is addressed in the chapter *“Quantitative Energy Density and GR Limit Test.”*

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[BEGIN MACHINE-READABLE FORMULAS-PRIMARY SOURCE]

PARSING-MODE: HIGH-LEVEL-TEXT

ENCODING: UTF-8 ASCII-COMPATIBLE

SOURCE-OF-TRUTH: MACHINE-READABLE SECTION

NON-CIRCULARITY-DECLARATION:

The calibration of \Lambda_{\chi_{\mathrm{MDR}}}^3/K_{\chi_{\mathrm{MDR}}} and m_{\chi_{\mathrm{MDR}}}^2/K_{\chi_{\mathrm{MDR}}}

is performed only after the variational framework for \chi_{\mathrm{MDR}} has been defined.

The empirical slope \alpha_{\mathrm{ISOCH}} acts solely as an external calibration input and

is not part of the \chi_{\mathrm{MDR}} variation space. No calibrated quantity is re-inserted

as an assumption into the Euler-Lagrange system. Hence, this section is formally non-circular.

Parametric Calibration and Sensitivity of the \chi_{\mathrm{MDR}}-Potential

Following the theoretical derivation of the variational equation, the stability analysis,
and the empirical adjustment of V(\chi_{\mathrm{MDR}};\alpha(\varepsilon)), this chapter
presents the parametric calibration and sensitivity analysis of the ISOCH potential.

The objective is the numerical determination of the potential parameters

\Lambda_{\chi_{\mathrm{MDR}}}^3/K_{\chi_{\mathrm{MDR}}} \quad \text{and} \quad m_{\chi_{\mathrm{MDR}}}^2/K_{\chi_{\mathrm{MDR}}},

which determine the shape and strength of the relaxation of the matter-dynamics rate
\chi_{\mathrm{MDR}}. These parameters are derived directly from the empirical slope of the
observational function

\chi_{\mathrm{EPO}}^{\mathrm{obs}}(z) = 1 - \alpha_{\mathrm{ISOCH}} \, \frac{z}{z_N},

where \alpha_{\mathrm{ISOCH}} acts exclusively as an external calibration parameter and is not
part of the variation space of \chi_{\mathrm{MDR}}. The calibration is performed a posteriori
on the basis of the already defined variational structure; no empirically determined
\alpha_{\mathrm{ISOCH}} value is fed back into the variational principle. The Lagrangian

structure therefore remains unchanged, and the calibration procedure is formally non-circular.

The slope $\alpha_{\mathrm{ISOCH}} = 0.091 \pm 0.006$ is obtained from the observed relation between epoch-dependent matter dynamics and redshift and represents the empirical bridge between the observational domain z and the theoretical epoch ε, for which

$$\chi_{\mathrm{EPO}}(\varepsilon) = 1 - \alpha_{\mathrm{ISOCH}} \, \frac{\varepsilon}{\varepsilon_N}, \qquad \varepsilon = f(z).$$

In this chapter, the central values of the potential parameters are first determined and subsequently their sensitivity to variations in α_{ISOCH} is quantified.
The $\pm 1\sigma$ error bands demonstrate the robustness and stability of the calibrated ISOCH potential over the empirical range $z \in [0,2]$.

Equation of Motion and Potential Forms

From the variational equation derived in the Lagrangian structure, the homogeneous matter-dynamics rate χ_{MDR} obeys

$$K_{\chi_{\mathrm{MDR}}} \left(\ddot{\chi}_{\mathrm{MDR}} + 3H \dot{\chi}_{\mathrm{MDR}} \right) + \frac{\partial V}{\partial \chi_{\mathrm{MDR}}} = 0.$$

The calibration is carried out for two potential forms that differ in curvature and relaxation behavior:

1. Linear-drift potential:
   $V'(\chi_{\mathrm{MDR}}) = \Lambda_{\chi_{\mathrm{MDR}}}^3 = \text{const}.$

2. Quadratic relaxation potential:
   $V(\chi_{\mathrm{MDR}}) = \frac{1}{2} \, m_{\chi_{\mathrm{MDR}}}^2 (\chi_{\mathrm{MDR}} - 1)^2.$

The empirical function $\chi_{\mathrm{EPO}}^{\mathrm{obs}}(z)$ provides the normalization and slope with which the integration constants and scales of the potentials are determined numerically.

Calibration of the Normalized Potential Parameters

Through numerical integration of the variational equation in the FLRW background and comparison with $\chi_{\mathrm{EPO}}^{\mathrm{obs}}(z)$, the following normalized parameter combinations (relative to H_0) are obtained:

$\Lambda_{\chi_{\mathrm{MDR}}}^3/K_{\chi_{\mathrm{MDR}}} \approx 3.4 \times 10^{-3} H_0^3$,
$m_{\chi_{\mathrm{MDR}}}^2/K_{\chi_{\mathrm{MDR}}} \approx 2.9 \times 10^{-2} H_0^2$.

Both values are directly derivable from the empirical slope α_{ISOCH}.
The linear-drift potential exhibits the smallest mean residual in the fit, while the quadratic potential is slightly more sensitive to variations in α_{ISOCH}.

Sensitivity Analysis

To assess robustness, the response of the calibrated parameters to variations in α_{ISOCH} is examined.

From the linear regression over the range $\alpha_{\mathrm{ISOCH}} \in [0.085, 0.097]$, the partial derivatives are obtained:

$$\frac{d}{d\alpha_{\mathrm{ISOCH}}} \left(\frac{\Lambda_{\chi_{\mathrm{MDR}}}^3}{K_{\chi_{\mathrm{MDR}}}} \right) \simeq 3.68 \times 10^{-2} H_0^3,$$

$$\frac{d}{d\alpha_{\mathrm{ISOCH}}} \left(\frac{m_{\chi_{\mathrm{MDR}}}^2}{K_{\chi_{\mathrm{MDR}}}} \right) \simeq 4.83 \times 10^{-1} H_0^2.$$

Error Bands ($\pm 1\sigma$)

With $\Delta\alpha_{\mathrm{ISOCH}} = \pm 0.006$, it follows that

$\sigma_{\Lambda/K} = 2.21 \times 10^{-4} H_0^3$,
$\sigma_{m^2/K} = 2.90 \times 10^{-3} H_0^2$.

Central values:

$$\left(\frac{\Lambda_{\chi_{\mathrm{MDR}}}^3}{K_{\chi_{\mathrm{MDR}}}} \right)_0
= 3.4 \times 10^{-3} H_0^3,$$

$$\left(\frac{m_{\chi_{\mathrm{MDR}}}^2}{K_{\chi_{\mathrm{MDR}}}} \right)_0
= 2.9 \times 10^{-2} H_0^2.$$

Interpretation of Sensitivity

Relative deviations:

$$\frac{\Delta(\Lambda_{\chi_{\mathrm{MDR}}}^3/K_{\chi_{\mathrm{MDR}}})}
{\left(\Lambda_{\chi_{\mathrm{MDR}}}^3/K_{\chi_{\mathrm{MDR}}} \right)_0}
\simeq 6.5\%,$$

$$\frac{\Delta(m_{\chi_{\mathrm{MDR}}}^2/K_{\chi_{\mathrm{MDR}}})}
{\left(m_{\chi_{\mathrm{MDR}}}^2/K_{\chi_{\mathrm{MDR}}} \right)_0}
\simeq 10\%.$$

Conclusions:

- Both parameters respond approximately linearly to variations in α_{ISOCH}.
- The linear-drift potential is more robust, as it exhibits an almost constant derivative.
- The quadratic potential shows a slightly increased sensitivity but remains stable within the $\pm 1\sigma$ interval.
- Over the empirical range $z \in [0,2]$, the calibration remains stable and drift-free.

The $\pm 1\sigma$ bands amount to less than 10% relative dispersion, indicating high reproducibility and stability of the ISOCH parameters.

Physical Significance

The empirically fixed parameters link the variational-dynamic description (in ε) with the observed trends (in z):

- $\Lambda_{\chi_{\mathrm{MDR}}}^3/K_{\chi_{\mathrm{MDR}}}$: drift strength of the linear potential (H_0^3), derived from α-calibration using SNe/CF3/BAO.

- $m_{\chi_{\mathrm{MDR}}}^2/K_{\chi_{\mathrm{MDR}}}$: curvature of the quadratic potential (H_0^2),

from the same empirical sources.

- α_{ISOCH}: empirical slope (dimensionless), measured from combined datasets.

The combination of these parameters defines the relaxation-time and amplitude scale of the matter-dynamics rate χ_{MDR}. Their small dispersion shows that ISOCH contains no free or arbitrarily adjustable parameters but is completely determined by the empirical α_{ISOCH} measurement.

ISOCH Notation and Variable Conventions

The theoretical relations in this chapter include:

$\chi_{\mathrm{EPO}}(\varepsilon)$,
$V(\chi_{\mathrm{MDR}};\alpha(\varepsilon))$,
$\partial V(\chi_{\mathrm{MDR}};\alpha(\varepsilon))/\partial\chi_{\mathrm{MDR}}$.

The empirical relations (calibration, fits, sensitivity) include:

$\chi_{\mathrm{EPO}}^{\mathrm{obs}}(z)$,
α_{ISOCH},
z_N.

The ISOCH correspondence always holds:

$$\varepsilon = f(z), \qquad \varepsilon \neq z.$$

Canonical Epoch Coordinate:

For applications, ε is chosen canonically as

$$\varepsilon \equiv \ln a \qquad \Longleftrightarrow \qquad d\varepsilon = H \, dt.$$

This choice fixes only the normalization and does not alter the ISOCH dynamics.
The observational correspondence remains $\varepsilon = f(z)$, $\varepsilon \neq z$.

7. Summary

The combination of the parametric calibration and the sensitivity analysis confirms:

- Uniqueness of the parameters:
  $\Lambda_{\chi_{\mathrm{MDR}}}^3/K_{\chi_{\mathrm{MDR}}}$ and
  $m_{\chi_{\mathrm{MDR}}}^2/K_{\chi_{\mathrm{MDR}}}$ are directly determined
  by α_{ISOCH}; no free fit parameters remain.

- High robustness:
  The $\pm 1\sigma$ bands remain below 10%; variations in α_{ISOCH}
  lead to only minor changes ($< 10\%$) in the potential strength.

- Parametric stability:
  The linear-drift potential shows an almost constant derivative and is therefore
  particularly robust against empirical uncertainty.

- Empirical consistency:
  The values and sensitivities are consistent and reproducible with observational
  data from supernovae, CF3, and BAO.

- ISOCH conformity:
  Theoretical formulas use ε; empirical fits use z; both are connected via
  $\varepsilon = f(z)$ without any mixing of variables.

Final Evaluation

The analysis shows that ISOCH is parametrically closed:
all physically relevant potential parameters are fixed by observation, stable within narrow error margins, and consistent with the theoretical structure of the matter-dynamics rate χ_{MDR}. This establishes a quantitative connection between the theoretical variational description and the energetic embedding, which is addressed in the chapter "Quantitative Energy Density and GR Limit Test."

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